Conservation of momentum in a central elastic collision with the demonstration track and timer 4 – 4

(Item No.: P1199605)

Curricular Relevance

Area of Expertise:	Education Level:	Topic:	Subtopic:	Experiment:
Physik	Klasse 7-10	Mechanik	Lineare Bewegung	

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Intermediate	10 Minutes	20 Minutes	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

Elastic collision, impulse, conservation of momentum, momentum transfer, conservation of energy, kinetic energy, coefficient of restitution

Overview

Introduction

An impulse acting on an object is defined as the change in momentum caused by a force F over a short period of time t. The momentum p is defined as the product of force and time. It is conserved, provided that there is no friction and that the collision is elastic. This means that, although the objects in a closed system with several objects can transfer or receive momenta, the total momentum of the system remains constant over time and in terms of its value, which in turn means that the energy is conserved.

Educational objective

If two carts collide elastically, they both transfer a momentum to the respective other cart and then continue to move with changed momenta. The direction of motion may be reversed, but the total momentum of the system prior to the collision is conserved:

$$p_1 + p_2 \ = \ p_1' + p_2' \ .$$

If the collision is completely elastic, the kinetic energy of the system is also conserved:

$$E_{\text{kin}} \ = \ E_{\text{kin}}' \ .$$

Related topics

Experiment P1199705 "Conservation of momentum in a central inelastic collision" can be performed for comparison. In addition, experiment P1199805 "Conservation of momentum in multiple central elastic collisions" demonstrates elastic collisions with a third cart.

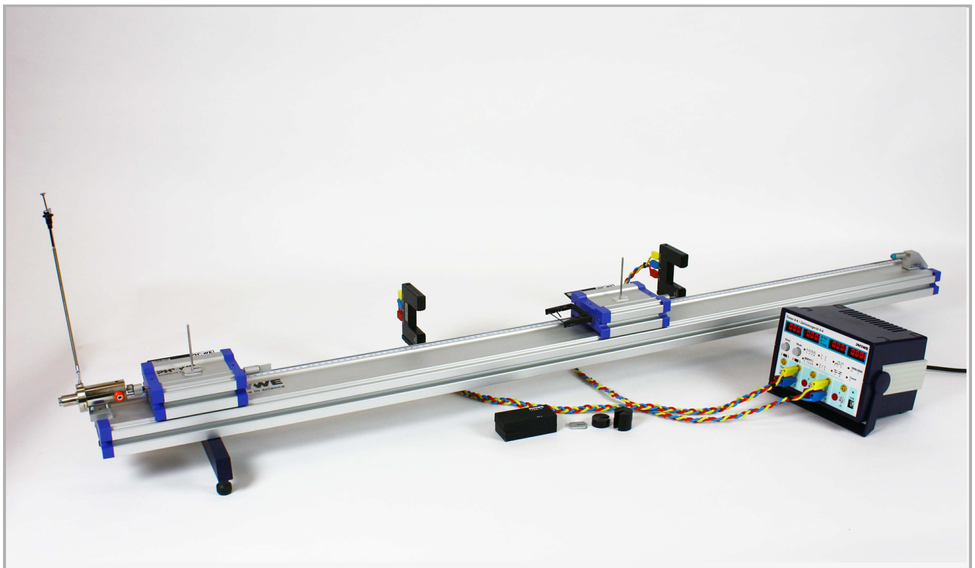

Figure 1: Experiment set-up

Equipment

Position No.	Material	Order No.	Quantity
1	Timer 4-4	13604-99	1
2	Starter system for demonstration track	11309-00	1
3	Demonstration track, aluminium, 1.5 m	11305-00	1
4	Cart, low friction sapphire bearings	11306-00	2
5	Light barrier, compact	11207-20	2
6	Portable Balance, OHAUS CS2000E	48911-00	1
7	End holder for demonstration track	11305-12	1
8	Weight for low friction cart, 400 g	11306-10	2
9	Magnet w.plug f.starter system	11202-14	1
10	Shutter plate for low friction cart, width: 100 mm	11308-00	2
11	Plate with plug	11202-10	1
12	Needle with plug	11202-06	1
13	Fork with plug	11202-08	1
14	Tube with plug	11202-05	1
15	Slotted weight, black, 10 g	02205-01	4
16	Slotted weight, black, 50 g	02206-01	3
17	Holder for light barrier	11307-00	2
18	Connecting cord, 32 A, 1000 mm, red	07363-01	2
19	Connecting cord, 32 A, 1000 mm, yellow	07363-02	2
20	Connecting cord, 32 A, 1000 mm, blue	07363-04	2
21	Slotted weight, silver bronze, 10 g	02205-02	4
22	Slotted weight, silver bronze, 50 g	02206-02	3
23	Plasticine, 10 sticks	03935-03	1
24	Slotted weight, blank, 1 g	03916-00	
25	Rubber bands for fork with plug, 10 pcs	11202-09	1

Tasks

1. Determination of the momenta before and after an elastic collision between a moving cart and a cart at rest.
2. Determination of the momenta before and after an elastic collision between two moving carts with the same direction of motion.
3. Determination of the momenta before and after an elastic collision between two moving carts with opposite directions of motion.

Set-up and procedure

Set-up

Set the experiment up as shown in Figure 1:

1. Precisely align the track horizontally by way of the three adjusting screws at the track bases.
2. Position the starter system at the left end of the track. Please note that, in order to start the cart with an initial momentum, the starter system must be installed so that the cart receives an impulse from the ram of the starter system (Fig. 2).

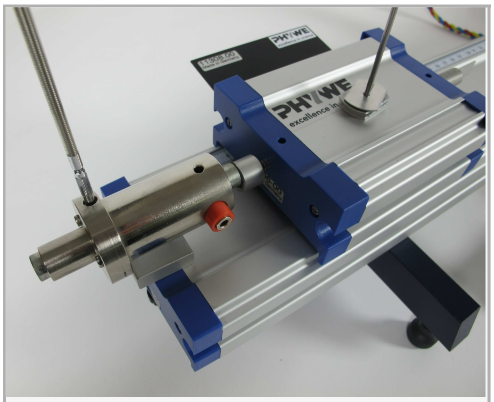

Fig. 2: Starter system for providing the necessary impulse

3. Attach a plasticine-filled tube to the end holder at the right-hand end of the track in order to stop the cart without a strong impact (Fig. 3).

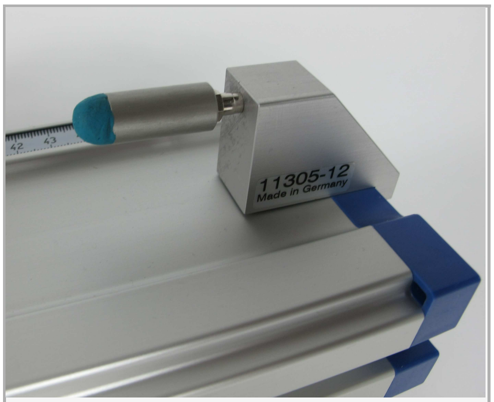

Fig. 3: End holder with plasticine

4. Fasten the two light barriers in the light barrier holders and position them at the 50 cm mark and 100 cm mark on the track. The light barrier that is closer to the starter system is light barrier 1, and the other one is light barrier 2.
5. Connect light barrier 1 to the sockets in field "1" and light barrier 2 to the sockets in field "3" of the timer. In doing so, connect the yellow sockets of the light barriers to the yellow sockets of the measuring instrument, the red sockets to their red counterparts, and the blue sockets of the light barriers to the white sockets of the timer (see Fig. 4).

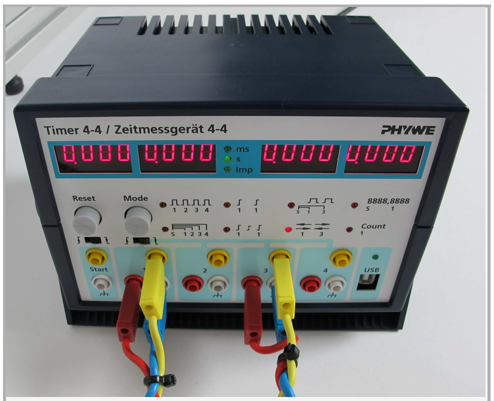

Fig. 4: Connection of the light barriers

6. In order to select the triggering edge, push the slide switch above field "1" of the timer to the right, i.e. to "falling edge" ( ).
7. Place the two carts on the track.
   - Equip the cart on the left, which is closer to the starter system (hereinafter referred to as cart 1 with the velocity v_1), with a magnet with a plug facing the starter system and with a plate with a plug in the direction of motion (see Fig. 5).

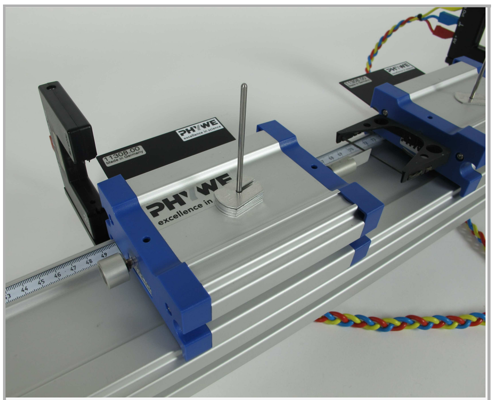

Figure 5: Elastic collision after the first light barrier

   - Equip the cart on the right (cart 2 with v_2) with the fork and rubber band facing cart 1 and with the needle with a plug facing the end holder (see Fig. 6).

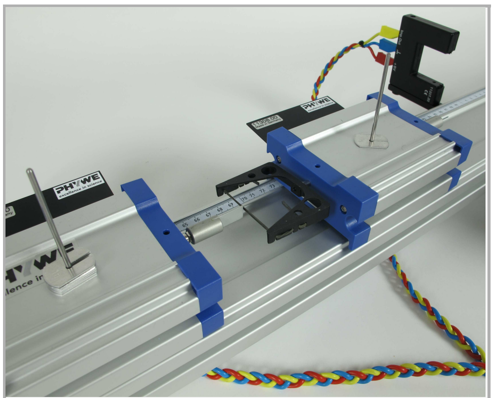

Figure 6: Completed collision prior to reaching the second light barrier

   - Fasten a shutter plate (w = 100 mm) in both carts on the side where the light barriers are located.

Procedure

1. In order to perform the measurements, set the timer to mode 6 "collision" (). Only the control inputs 1 and 3 are active in this case. Measure up to two shading times for each of the light barriers. The times of interruption of light barrier 1 are indicated on the first two displays, and the ones of light barrier 2 on the last two displays. The first interruption of a light barrier is indicated on the respective display on the left, the second one on the respective display on the right.
2. Prior to starting the measurement, determine the masses of the carts by way of the balance. Small corrections (especially if both cart masses are to be identical) can be realised with the aid of the 1 g weights as shown in Fig. 6.
3. Prior to every collision experiment, press the "Reset" button in order to reset the displays.
4. Use the shading times ti and the shutter plate length w = 100 mm to determine the velocities $v_i = w/t_i$. Since the velocities are vector quantities, their sign is important. All of the velocities that are opposite to v_1 have the opposite sign of v_1.
5. In order to be able to distinguish the measurement data of the two carts, the shading times prior to the collision are named t_1 and t_2 and the ones after the collision are named t'_1 and t'_2. The same nomenclature is to be used for the resulting calculated velocities and momenta.

a) Cart 1 in motion, cart 2 at rest:

1. Light barrier 1 should be located at approximately 50 cm and light barrier 2 at 100 cm.
2. Position cart 1 in the starter system and cart 2 between the two light barriers. Ensure that cart 1 has completely passed through the light barrier before it touches cart 2 (compare Fig. 5). In addition, the collision must be completed before the shutter plate of cart 2 reaches light barrier 2 (compare Fig. 6).
3. The starter system accelerates cart 1 in the direction of cart 2. During this process, it receives an initial velocity v_1 and collides with cart 2, which then moves with the velocity v_2. If the carts have different masses, cart 1 will follow cart 2 with the velocity v1' or it will be reflected. The displays of the timer indicate the following values from left to right: $(t_1/-/t'_2/t'_1)$ or $(t_1/t'_1/t'_2/-)$.
4. Repeat the measurement for different cart masses and mass ratios.

b) Cart 1 and cart 2 in motion, same direction:

1. Light barrier 1 should be located at approximately 30 cm and light barrier 2 at 100 cm.
2. Position cart 1 in the starter system and cart 2 between the two light barriers.
3. Transfer a small impulse to cart 2 by hand in the direction of the end holder before cart 1 is set into motion by the starter system. The collision must be completed before cart 2 reaches light barrier 2 $(v_1 > v_2)$. The displays of the timer indicate the following values from left to right: $(t_1/-/t'_2/t'_1)$.
4. Repeat the measurement for different cart masses and mass ratios.

c) Cart 1 and cart 2 in motion, opposite direction:

1. Light barrier 1 should be located at approximately 40 cm and light barrier 2 at 100 cm.
2. Position the carts on opposite ends of the track beyond the light barriers. They both receive an initial velocity. To do so, both carts can be pushed by hand, or cart 1 can be accelerated by the starter system. Alternatively, the end holder can be replaced with a second starter system so that the experiment can be performed under reproducible conditions.
3. Ensure that the two carts do not collide until they have completely passed through the light barriers. After the collision, the carts move apart from one another, once again passing through the light barriers. The displays of the timer indicate the following values from left to right: $(t_1/t'_1/t_2/t'_2)$.
4. Repeat the measurement for different cart masses and mass ratios.

Evaluation

The momentum p of an object with the mass m can be calculated based on its velocity v:

$$p = m \cdot v \, . \quad (1)$$

In the case of a central collision of two masses m_1 and m_2 with the momenta $p_1 = m_1 \cdot v_1$ and $p_2 = m_2 \cdot v_2$ before the collision and $p'_1 = m'_1 \cdot v'_1$ and $p'_2 = m'_2 \cdot v'_2$ after the collision, the law of conservation of momentum applies:

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \, . \quad (2)$$

If the velocity v_2 prior to the collision is unknown, it can be calculated based on the other velocities and masses:

$$v_2 = \frac{m_1 v'_1 + m_2 v'_2 - m_1 v_1}{m_2} \, . \quad (3)$$

In the case of elastic collisions, not only the total momentum p but also the kinetic energy E_{kin} of the total system is conserved. The law of conservation of the kinetic energy before and after the collision is as follows:

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 {v_1}'^2 + \frac{1}{2} m_2 {v_2}'^2 \, . \quad (4)$$

With the two laws of conservation (2) and (4), the velocities after the collision v'_1 and v'_2 can be calculated based on the initial velocities v_1 and v_2:

$$v'_1 = \frac{(m_1 - m_2) v_1 + 2 m_2 v_2}{m_1 + m_2} \, , \quad (5)$$

$$v'_2 = \frac{(m_2 - m_1) v_2 + 2 m_1 v_1}{m_1 + m_2} \, . \quad (6)$$

Based on the equations (5) and (6), the following results for the difference between the velocities:

$$v'_2 - v'_1 = v_1 - v_2 \quad (7)$$

The difference can be considered as a relative velocity with which cart 1 and cart 2 approach one another or move apart.
The relative velocity before and after the collision is identical. In the experiment, the collisions are never completely elastic so that the law of conservation of kinetic energy is affected. As a consequence, equations (5)–(7) are not absolutely valid. It is now possible to introduce the coefficient of restitution δ, which is a measures of the elasticity of the collision:

$$\delta = \frac{v'_1 - v'_2}{v_2 - v_1} \, . \quad (8)$$

In the case of a completely elastic collision, the value of this coefficient of restitution is 1. In the case of an inelastic collision, its value is 0. With this, equations (5) and (6) can be transformed into

$$v'_1 = \frac{(m_1 - \delta m_2) v_1 + (1+\delta) m_2 v_2}{m_1 + m_2} \text{ und} \quad (9)$$

$$v'_2 = \frac{(m_2 - \delta m_1) v_2 + (1+\delta) m_1 v_1}{m_1 + m_2} \, . \quad (10)$$

A comparison of the calculated velocities with the measurement values in Tables 1–3 and 7–9 shows that they match very well while taking into account inelastic effects due to the coefficient of restitution.

a) Cart 1 in motion, cart 2 at rest:

1. $m_1 = m_2$: If the masses of the two carts are identical, cart 1 transfers its momentum p_1 completely to cart 2 and stops (see Table 1).
2. $m_1 > m_2$: If the mass of cart 1 is greater than the mass of the cart at rest, only part of the momentum is transferred. After the collision, cart 1 continues to move, but with a lower velocity than before the collision (see Table 2).
3. $m_1 < m_2$: If the mass of cart 1 is smaller than the mass of the cart at rest, the momentum transferred is higher than the momentum of cart 1 before the collision. Due to the conservation of momentum, cart 1 is reflected by 2 during the collision (see Table 3).

Table 1: Same direction, $m_1 = m_2, v_1 \neq 0, v_2 = 0.$

m_1 in kg	t_1 in s	v_1 in m/s	p_1 in kg·m/s	m_2 in kg	t_2' in s	v_2' in m/s	p_2' in kg·m/s
0.400	0.166	0.602	0.241	0.400	0.175	0.571	0.229
0.540	0.198	0.505	0.273	0.540	0.211	0.474	0.256
0.800	0.242	0.413	0.331	0.800	0.265	0.377	0.302

p_2'–p_1 in kg·m/s	E_{kin} in kg·m²/s²	E_{kin}' in kg·m²/s²	E_{kin}'–E_{kin} in kg·m²/s²	δ
−0.012	0.0726	0.0653	−0.0073	0.949
−0.017	0.0689	0.0606	−0.0082	0.938
−0.029	0.0683	0.057	−0.0113	0.913

Table 2: Same direction, $m_1 > m_2, v_1 \neq 0, v_2 = 0.$

m_1 in kg	t_1 in s	v_1 in m/s	p_1 in kg·m/s	t_1' in s	v_1' in m/s	p_1' in kg·m/s
0.800	0.174	0.575	0.460	0.535	0.187	0.150
1.000	0.172	0.581	0.581	0.411	0.243	0.243
1.200	0.159	0.629	0.755	0.327	0.306	0.367
m_2 in kg	t_2 in s	v_2 in m/s	p_2 in kg·m/s	t_2' in s	v_2' in m/s	p_2' in kg·m/s
0.400	–	–	–	0.135	0.741	0.296
0.400	–	–	–	0.126	0.794	0.317
0.400	–	–	–	0.111	0.901	0.360

(p_1'+p_2')–p_1 in kg·m/s	E_{kin} in kg·m²/s²	E_{kin}' in kg·m²/s²	E_{kin}'–E_{kin} in kg·m²/s²	δ
−0.014	0.132	0.124	−0.008	0.964
−0.021	0.169	0.156	−0.013	0.947
−0.027	0.237	0.218	−0.019	0.946

Table 3: Same direction, $m_1 < m_2, v_1 \neq 0, v_2 = 0.$

m_1 in kg	t_1 in s	v_1 in m/s	p_1 in kg·m/s	t_1' in s	v_1' in m/s	p_1' in kg·m/s
0.400	0.160	0.625	0.250	0.697	−0.143	−0.057
0.400	0.161	0.621	0.248	0.462	−0.216	−0.087
0.400	0.162	0.617	0.247	0.392	−0.255	−0.102
m_2 in kg	t_2 in s	v_2 in m/s	p_2 in kg·m/s	t_2' in s	v_2' in m/s	p_2' in kg·m/s
0.800	–	–	–	0.252	0.397	0.317
1.000	–	–	–	0.299	0.334	0.334
1.200	–	–	–	0.342	0.292	0.351

$(p_1'+p_2')–p_1$ in kg·m/s	E_{kin} in kg·m²/s²	E_{kin}' in kg·m²/s²	$E_{kin}'–E_{kin}$ in kg·m²/s²	δ
0.010	0.078	0.067	−0.011	0.864
−0.001	0.077	0.065	−0.012	0.887
0.002	0.076	0.064	−0.012	0.887

b) Cart 1 and cart 2 in motion, same direction:

1. If the velocities of the two carts have the same direction, the quicker cart 1 transfers a momentum to the slower cart 2.
2. $m_1 = m_2$: If the masses of the two carts are identical, they have exchanged their velocity values after the collision (see Table 4):
$$v_2' = v_1 \text{ and } v_1' = v_2 \ . \quad (11)$$
3. $m_1 > m_2$ and $m_1 < m_2$: During the collision, cart 1 transfers part of its momentum. Cart 2 moves with a higher velocity, while cart 1 reduces its velocity or even inverts its direction of motion (see Tables 5 and 6).
4. Calculate the momentum p_2 and the initial velocity v_2 based on the measurements and on equations (2) and (3).

Table 4: Same direction, $m_1 = m_2, v_1 \neq 0, v_2 \neq 0.$

m_1 in kg	t_1 in s	v_1 in m/s	p_1 in kg·m/s	t_1' in s	v_1' in m/s	p_1' in kg·m/s
0.400	0.114	0.877	0.351	0.286	0.350	0.140
0.540	0.114	0.877	0.474	0.283	0.353	0.191
0.800	0.108	0.926	0.741	0.233	0.429	0.343
m_2 in kg	t_2' in s	v_2' in m/s	p_2' in kg·m/s	t_2 in s	v_2 in m/s	p_2 in kg·m/s
0.400	0.117	0.855	0.342	–	0.327	0.131
0.540	0.116	0.862	0.466	–	0.338	0.183
0.800	0.110	0.909	0.727	–	0.412	0.330

Table 5: Same direction, $m_1 > m_2, v_1 \neq 0, v_2 \neq 0.$

m_1 in kg	t_1 in s	v_1 in m/s	p_1 in kg·m/s	t_1' in s	v_1' in m/s	p_1' in kg·m/s
0.600	0.175	0.571	0.343	0.330	0.303	0.182
0.800	0.239	0.418	0.335	0.424	0.236	0.189
1.200	0.186	0.538	0.645	0.280	0.357	0.429
m_2 in kg	t_2' in s	v_2' in m/s	p_2' in kg·m/s	t_2 in s	v_2 in m/s	p_2 in kg·m/s
0.400	0.163	0.613	0.245	–	0.211	0.084
0.400	0.210	0.476	0.190	–	0.111	0.044
0.400	0.150	0.667	0.267	–	0.125	0.050

Table 6: Same direction, $m_1 < m_2, v_1 \neq 0, v_2 \neq 0.$

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m_1 in kg	t_1 in s	v_1 in m/s	p_1 in kg·m/s	t_1' in s	v_1' in m/s	p_1' in kg·m/s
0.400	0.161	0.621	0.248	0.300	0.333	0.133
0.400	0.160	0.625	0.250	0.383	0.261	0.104
0.400	0.161	0.621	0.248	0.537	0.186	0.074
m_2 in kg	t_2' in s	v_2' in m/s	p_2' in kg·m/s	t_2 in s	v_2 in m/s	p_2 in kg·m/s
0.600	0.179	0.559	0.335	–	0.367	0.220
0.800	0.191	0.524	0.419	–	0.342	0.273
1.000	0.211	0.474	0.474	–	0.300	0.300

c) Cart 1 and cart 2 in motion, opposite direction:

1. m_1 = m_2: In the case of opposite directions of motion and equal masses, the carts invert their direction of motion during the collision and their velocity values are exchanged after the collision (see Table 7).
2. $m_1 > m_2$ and $m_1 < m_2$: In the case of unequal masses of the carts, they reflect one another and invert their respective direction of motion (see Tables 8 and 9).

Table 7: Opposite direction, $m_1 = m_2, v_1 > 0, v_2 < 0.$

m_1 in kg	t_1 in s	v_1 in m/s	p_1 in kg·m/s	t_1' in s	v_1' in m/s	p_1' in kg·m/s
0.400	0.218	0.459	0.183	0.238	−0.420	−0.168
0.540	0.253	0.395	0.213	0.310	−0.323	−0.174
0.800	0.241	0.415	0.332	0.279	−0.358	−0.287
m_2 in kg	t_2' in s	v_2' in m/s	p_2' in kg·m/s	t_2 in s	v_2 in m/s	p_2 in kg·m/s
0.400	0.202	−0.495	−0.198	0.241	0.415	0.166
0.540	0.248	−0.403	−0.218	0.285	0.351	0.189
0.800	0.208	−0.481	−0.385	0.313	0.319	0.256

$(p_1'+p_2')-(p_1+p_2)$ in kg·m/s	E_{kin} in kg·m²/s²	E_{kin}' in kg·m²/s²	$E_{kin}'-E_{kin}$ in kg·m²/s²	δ
0.012	0.091	0.070	−0.021	0.876
0.020	0.086	0.061	−0.025	0.843
0.022	0.161	0.092	−0.069	0.757

Table 8: Opposite direction, $m_1 > m_2, v_1 > 0, v_2 < 0.$

m_1 in kg	t_1 in s	v_1 in m/s	p_1 in kg·m/s	t_1' in s	v_1' in m/s	p_1' in kg·m/s
0.540	0.188	0.532	0.287	0.437	−0.229	−0.124
0.800	0.238	0.420	0.336	0.648	−0.154	−0.123
0.800	0.234	0.427	0.342	0.527	−0.190	−0.152
m_2 in kg	t_2' in s	v_2' in m/s	p_2' in kg·m/s	t_2 in s	v_2 in m/s	p_2 in kg·m/s
0.400	0.209	−0.478	−0.191	0.160	0.625	0.250
0.400	0.177	−0.565	−0.226	0.150	0.667	0.267
0.540	0.220	−0.455	−0.245	0.185	0.541	0.292

$(p_1'+p_2')-(p_1+p_2)$ in kg·m/s	E_{kin} in kg·m²/s²	E_{kin}' in kg·m²/s²	$E_{kin}'-E_{kin}$ in kg·m²/s²	δ
0.031	0.122	0.092	−0.030	0.845
0.033	0.134	0.098	−0.036	0.833
0.044	0.129	0.093	−0.036	0.828

Table 9: Opposite direction, $m_1 < m_2, v_1 > 0, v_2 < 0.$

m_1 in kg	t_1 in s	v_1 in m/s	p_1 in kg·m/s	t_1' in s	v_1' in m/s	p_1' in kg·m/s
0.400	0.217	0.461	0.184	0.168	−0.595	−0.238
0.400	0.216	0.463	0.185	0.188	−0.532	−0.213
0.540	0.249	0.402	0.217	0.184	−0.543	−0.293
m_2 in kg	t_2' in s	v_2' in m/s	p_2' in kg·m/s	t_2 in s	v_2 in m/s	p_2 in kg·m/s
0.540	0.189	−0.529	−0.286	0.365	0.274	0.148
0.800	0.287	−0.348	−0.279	0.594	0.168	0.135
0.800	0.217	−0.461	−0.369	0.508	0.197	0.157

(p_1'+p_2')–(p_1+p_2) in kg·m/s	E_{kin} in kg·m²/s²	E_{kin}' in kg·m²/s²	E_{kin}'–E_{kin} in kg·m²/s²	δ
0.011	0.118	0.091	−0.027	0.878
0.015	0.091	0.068	−0.024	0.863
0.016	0.128	0.095	−0.033	0.858

Note

1. In order to accelerate cart 1 with the starter system, the ram must be pushed in until it locks into place. Since the starter system offers three different levels, it must be ensured that the same locking position is used for all the experiments so that the same force is transferred.
2. Ensure that during the collision the rubber band is not pushed back to such an extent that the plate of one cart touches the fork of the other cart.
3. Low velocities combined with old rubber bands having only little spring force lead to higher energy losses.
4. The correct position of the shutter plates on the carts should be checked prior to every measurement, since they may be dislocated when the carts are abruptly stopped.
5. The plasticine should also be reshaped from time to time in order to buffer the impact of the cart to the highest possible extent.
6. The carts do not move completely without friction. There is still some residual friction and the total momentum decreases slightly by approximately 6 %. Another reason for the decrease in total momentum after the collision may be that the collision is not exactly central. This results in momentum components that are perpendicular to the track. However, these are not taken into consideration during the evaluation. In addition, the collision is not completely elastic: the kinetic energy decreases by up to 25 %.