

Equivalence of inertial mass and gravitational mass with the demonstration track and timer 4 - 4

(Item No.: P1199405)

Curricular Relevance

Area of Expertise: Physik	Education Level: Klasse 7-10	Topic: Mechanik	Subtopic: Lineare Bewegung	Experiment:
Difficulty  Intermediate	Preparation Time  20 Minutes	Execution Time  10 Minutes	Recommended Group Size  2 Students	
Additional Requirements:		Experiment Variations:		
Keywords: Gravitational and inertial mass, equivalence, accelerated motion, law of gravitation, Newton's axioms				

Overview

Introduction

The mass of an object can be determined in different ways: by weighing the gravitational mass or by measuring the acceleration in the case of a known and constant accelerating force, which yields the inertial mass. This is based on the assumption that the gravitational mass and inertial mass are equivalent.

In this experiment, a cart with different gravitational masses is accelerated on the demonstration track. Its inertial mass is determined and the relationship between gravitational and inertial mass is derived.

Educational objective

So far, the term mass has been used in the context of the weight of an object and its motion. However, this term actually describes two different characteristics: the gravitational mass and the inertial mass. Due to the fact that they are equivalent, they are usually not differentiated, but the aim of this experiment is to put an emphasis on the fact that they are two fundamentally independent characteristics.

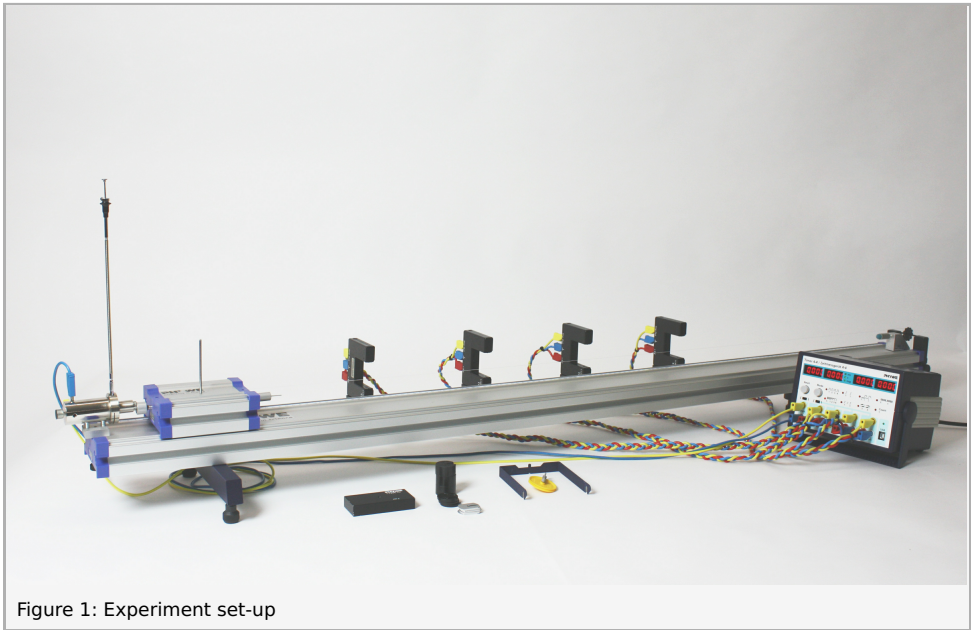


Figure 1: Experiment set-up

Equipment

Position No.	Material	Order No.	Quantity
1	Timer 4-4	13604-99	1
2	Starter system for demonstration track	11309-00	1
3	Demonstration track, aluminium, 1.5 m	11305-00	1
4	Cart, low friction sapphire bearings	11306-00	1
5	Light barrier, compact	11207-20	4
6	Portable Balance, OHAUS CS2000E	48911-00	1
7	End holder for demonstration track	11305-12	1
8	Weight for low friction cart, 400 g	11306-10	1
9	Magnet w.plug f.starter system	11202-14	1
10	Holder for pulley	11305-11	1
11	Pulley for demonstration track	11305-10	1
12	Shutter plate for low friction cart, width: 100 mm	11308-00	1
13	Needle with plug	11202-06	1
14	Weight holder, silver bronze, 1 g	02407-00	1
15	Tube with plug	11202-05	1
16	Slotted weight, black, 50 g	02206-01	3
17	Slotted weight, black, 10 g	02205-01	4
18	Holder for light barrier	11307-00	4
19	Connecting cord, 32 A, 1000 mm, red	07363-01	4
20	Connecting cord, 32 A, 1000 mm, yellow	07363-02	5
21	Connecting cord, 32 A, 1000 mm, blue	07363-04	5
22	Plasticine, 10 sticks	03935-03	1
23	Slotted weight, blank, 1 g	03916-00	
24	Silk thread, l = 200 m	02412-00	1

Tasks

1. Determination of the total gravitational mass and of the driving mass by weighing.
2. Determination of the acceleration of the cart and, thereby, of the inertial mass of the system.
3. Comparison of the gravitational mass and inertial mass.

Set-up and procedure

Set-up

Set the experiment up as shown in Figure 1:

1. In order to compensate for slight friction effects, the track must be slightly inclined by way of the adjusting screws at the track bases so that the cart is still just about prevented from rolling to the right.
2. Position the starter system at the left end of the track. Please note that, in order to start the cart without an initial momentum, the starter system must be installed so that the ram moves away from the cart when the starter system is triggered (Fig. 2).

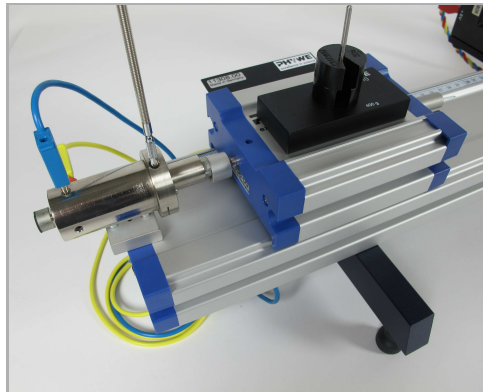


Fig. 2: Starter system without an impulse

3. Attach a plasticine-filled tube to the end holder at the right-hand end of the track in order to stop the cart without a strong impact (see Fig. 3).

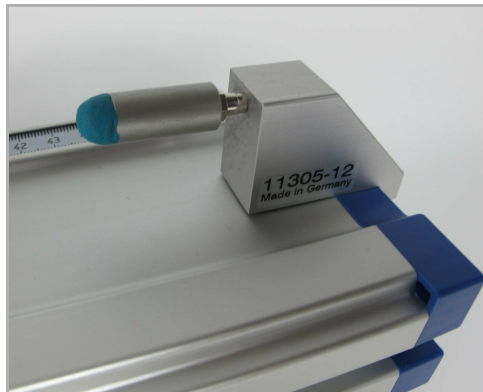


Fig. 3: End holder with plasticine

4. Install the pulley with the holder for the pulley at the right-hand end of the track and add the incremental wheel.
5. Equip the cart with the magnet with a plug and with the shutter plate ($w = 100 \text{ mm}$).
6. Insert the end of the thread from above through the vertical hole of the end cap of the cart and secure it in place by plugging the needle with a plug into the front (see Fig. 4).

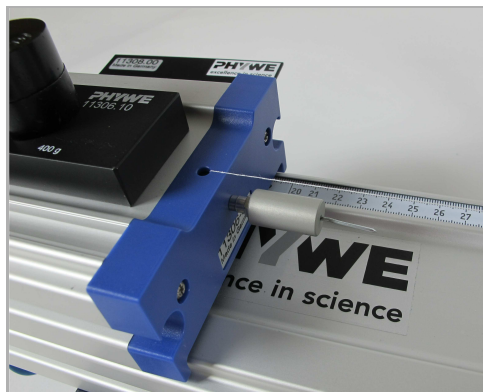


Fig. 4: Fastening of the thread on the cart

7. Lay the thread over the incremental wheel of the light barrier and knot its end onto the weight holder so that the latter is suspended freely just below the wheel (see Fig. 5). The force exerting the constant acceleration is the weight holder with 5 to 20 weights (1 g each) placed on it. Ensure that the thread is parallel to the track.

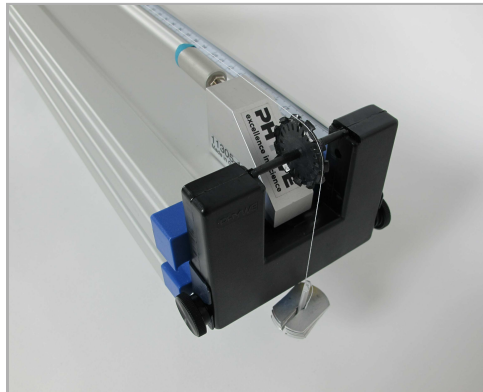


Fig. 5: Positioning of the weight holder

8. Install the four light barriers on the track by way of the light barrier holders and distribute them evenly over the track. Ensure that the back part of the shutter plate on the moving cart can pass through all of the light barriers (see Fig. 6) before the weight holder touches the floor.

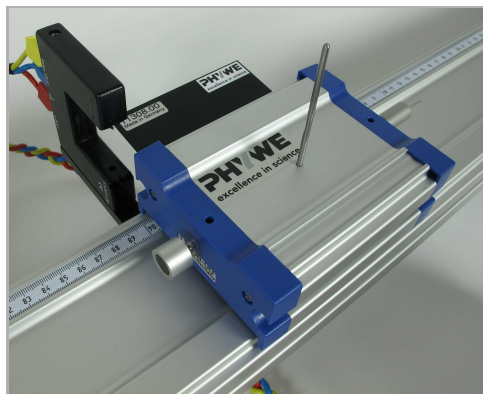


Fig. 6: Release of a light barrier following the passing of the shutter plate

9. Connect the four light barriers from the left to the right to the sockets in the fields "1" to "4" of the timer. In doing so, connect the yellow sockets of the light barriers to the yellow sockets of the measuring instrument, the red sockets to their red counterparts, and the blue sockets of the light barriers to the white sockets of the timer (see Fig. 7).

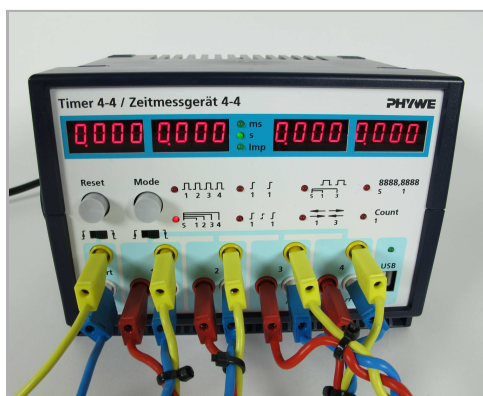


Fig. 7: Connection of the light barriers and starter system

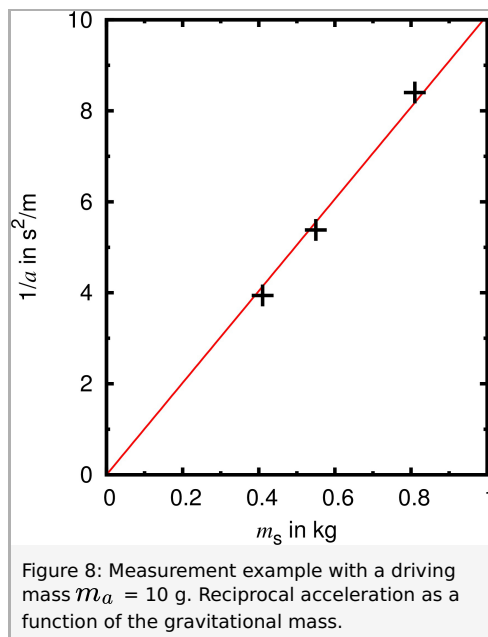
10. Connect the starter system to the two "Start" sockets of the timer. Ensure that the polarity is correct. Connect the red socket of the starter system to the yellow socket of the timer.
11. In order to select the triggering edge, push the two slide switches of the timer to the right, i.e. to "falling edge" (▼)

Procedure

1. The cart is released by the starter system and it undergoes constant acceleration until the weight holder touches the floor. Then, it continues to move at a constant velocity.
2. First, measure the times $t_1 \dots t_4$, from the start up to reaching the corresponding light barriers in mode 2 (). Then, perform one measurement in mode 1 () in order to determine the corresponding velocities. During this measurement, the shading times $\Delta t_1 \dots \Delta t_4$ of the four light barriers are determined. These are then used in order to calculate the average velocity of the cart passing through the light barriers based on the length of the shutter plate (100 mm).
3. Repeat the time measurement up to five times for different cart masses but with a constant mass of the weight holder. Prior to every recording process, press the "Reset" button in order to reset the display. Prior to the start of a new series of measurements, determine the total gravitational mass $m_s = m_W + m_A$ (cart mass and accelerating mass of the driving object).
4. The gravitational mass of an object is proportional to the acting weight force F_g . It must be determined by way of a balance for the driving object (m_A , weight holder plus weights).

Evaluation

1. Use the five measurements of $t_1 \dots t_4$ and $\Delta t_1 \dots \Delta t_4$ for a specific cart mass to calculate the mean values $t_{1m} \dots t_{4m}$ and $\Delta t_{1m} \dots \Delta t_{4m}$.
2. The shutter plate length $w = 100 \text{ mm}$ is used as the basis for determining the mean velocities $v_{im}(t_{im}) = b / \Delta t_{im}$ and the accelerations $a_i = v_{im}(t_{im}) / t_{im}$.
3. Take the mean of the four accelerations a_i that are part of same series of measurements. The resulting values a_m are then used to determine the reciprocal values of the acceleration $1/a_m$ (see Table 1).
4. Finally, plot the reciprocal accelerations $1/a_m$ against the total gravitational mass m_g (cart mass and driving mass) in a system of coordinates and determine the gradient of the straight line through these points. For comparison, the values of the measurement example are shown in Figure 8.



5. Every object has gravity. Due to the gravitational pull of the Earth, it is subject to an accelerating force, the weight force F_g , which is proportional to its gravitational mass m_g (Newton's law of gravitation). In this experiment, the gravitational mass $m_s = m_W + m_A$ comprises the mass of the cart and of the driving object.
6. If no force acts upon an object, the object will remain in its state of motion (Newton's first law). If an external force acts upon the object, the object will resist the change of its state of motion. This characteristic is known as its inertia. The greater its inertial mass m_i is, the more force must be applied in order to accelerate the object to a certain value (Newton's second law).
7. The values of the $(1/a, m_s)$ system of coordinates in Fig. 8 result in a straight line through the origin. In the case of a constant accelerating force, the gravitational force of the system is inversely proportional to the acceleration of the cart: $m_s \propto \frac{1}{a}$ for $F_g = \text{const}$.
The greater the gravitational mass m_s is, the smaller the acceleration a will become.
8. In accordance with Newton's second law,
$$F_g = m_t \cdot a$$

the inertial mass m_i of an object is inversely proportional to its acceleration a . If both the gravitational mass and the inertial mass are inversely proportional to the acceleration, the gravitational mass and the inertial mass must be proportional to each other:
$$m_s = k \cdot m_t.$$

The proportionality factor k is determined with the aid of a gradient triangle on the graph that is shown in Figure 8. The gradient has a value of $10.1 \text{ s}^2/(\text{kg} \cdot \text{m})$. Its reciprocal value of $0.0990 \text{ kg} \cdot \text{m/s}^2$ corresponds to the weight force $F_g = 0.0981 \text{ kg} \cdot \text{m/s}^2$ (see Table 1). This means that the gravitational mass and the inertial mass must be identical:
$$m_s = m_t.$$

Table 1: Measurement example

Light barrier 1					
F_g in N	m_s in kg	t_{1m} in s	Δt_{1m} in s	v_1 in m/s	a_1 in m/s ²

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0.0981	0.410	1.235	0.293	0.341	0.276
0.0981	0.550	1.436	0.344	0.291	0.202
0.0981	0.810	1.781	0.430	0.233	0.131
Light barrier 2					
F_g in N	m_s in kg	t_{2m} in s	Δt_{2m} in s	v_2 in m/s	a_2 in m/s ²
0.0981	0.410	1.776	0.220	0.454	0.256
0.0981	0.550	2.067	0.258	0.387	0.187
0.0981	0.810	2.573	0.323	0.309	0.120
Light barrier 3					
F_g in N	m_s in kg	t_{3m} in s	Δt_{3m} in s	v_3 in m/s	a_3 in m/s ²
0.0981	0.410	2.202	0.185	0.541	0.245
0.0981	0.550	2.566	0.217	0.460	0.179
0.0981	0.810	3.200	0.273	0.367	0.115
Light barrier 4					
F_g in N	m_s in kg	t_{4m} in s	Δt_{4m} in s	v_4 in m/s	a_4 in m/s ²
0.0981	0.410	2.555	0.163	0.613	0.240
0.0981	0.550	2.980	0.192	0.520	0.175
0.0981	0.810	3.722	0.243	0.412	0.111
Average values					
F_g in N	m_s in kg	a_m in m/s ²	$1/a$ in s ² /m		
0.0981	0.410	0.254	3.94		

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0.0981	0.550	0.186	5.38
0.0981	0.810	0.119	8.40

Note

1. In order to decrease the distance between the weight holder and the incremental wheel, the thread can be shortened by turning the needle with a plug on the cart several times, thereby winding the thread up.
2. Strictly speaking, the velocities v_i that are calculated based on Δt_i , are not instantaneous velocities, since the cart is still subject to acceleration when the shutter plate passes through the light barrier. Consequently, the velocities result from a secant gradient and not from a tangent gradient of the graph of $s(t)$. With $\Delta s = 0.1$ m, a systematic error of approximately 2 % must be taken into consideration.